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Computer-Program Model for Predicting Horizontally and Vertically Polarized VLF Atmospheric Radio Noise at Elevated Receivers

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A computer program has been developed that can predict horizontally and vertically polarized atmospheric radio noise at any altitude or location in the earth-ionosphere waveguide in the very-low-frequency (VLF) range from 10 to 30 kHz. The new program, HORNS, uses the outputs of two previously written programs, COMPWR and NOISLAN, which predict the vertical electric noise field at the ground. The HORNS program computes all the field components at any altitude using the vertical electric field at the earth's surface as a basis. Predicted values from several versions

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20. ABSTRACT (Continued)

of the new model have been compared with presently available data. The results are encouraging, but more data are needed to test the model.

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EXECUTIVE SUMMARY

Introduction

The Defense Communications Agency (DCA) has been evaluating the effectiveness of horizontal electric polarized waves for high-altitude air-to-air communication. Horizontal polarization is the best choice because a high-speed high-altitude jet aircraft can trail a horizontal long wire antenna, which preferentially excites transverse-electric (TE) radio waves, and because the ambient noise caused by the vertically polarized lightning strokes, which is most of the ambient noise in VLF radio systems, is believed to be least at horizontal polarization. A computer-program model has been developed for predicting the horizontally and vertically polarized atmospheric noise at any location, season, time of day, and altitude. This noise model will play a role in the prediction of VLF system performance.

The Program For The Atmospheric-Noise Model

The program for the atmospheric-noise model combines two previously written programs called COMPWR and NOISLAN with a new program called HORNS. The COMPWR and NOISLAN programs, produced by the Westinghouse Georesearch Laboratory (WGL) under contract to NRL, predict the vertically polarized electric atmospheric noise in the frequency band from 10 to 30 kHz in the quasi-transverse-magnetic (quasi-TM) waveguide modes at the surface of the earth at any location, time of day, or season. The HORNS program calculates the horizontally and vertically polarized electric and magnetic fields at any altitude in the earth-ionosphere waveguide, when the vertical electric fields in the propagating modes at the earth's surface are known. The predictions of the COMPWR and NOISLAN programs compare well with extensive measurements of worldwide atmospheric noise. The predictions of the algorithms of the HORNS program for the high-altitude fields have been compared with experimental data to some extent, and they should be reasonably accurate, since they are based on full-wave solutions of Maxwell's equations. The COMPWR program, which calculates the vertical effective radiated power (VERP) for each noise source, has two available options. COMPWR option I is the original WGL model. COMPWR option II contains different noise-source powers and diurnal power-modifier functions that vary the VERP of each atmospheric noise source over a 24-hour period. The NOISLAN program, which calculates propagation effects and sums the field strength contributions from all the sources, also has two available options. Option I is the original WGL model. Option II contains isotropic-waveguide-model attenuation rates and excitation factors as computed by a program called WAVEGUID using exponential ionosphere profiles with $\beta = 0.3\ \mathrm{km}^{-1}$ and $h = 70$ km for day and with $\beta = 0.5\ \mathrm{km}^{-1}$ and $h = 90$ km for night. The HORNS program calculates components of the high-altitude electromagnetic noise field propagated by three quasi-TM modes in the earth-ionosphere waveguide. The noise propagated in quasi-TE modes is not evaluated.

Comparison With Data

High-altitude noise data available for analyzing the HORNS model are limited. The Mitre Corporation has made a few airborne noise measurements suitable for comparison with the atmospheric noise predicted by the HORNS computer program. Reasonably good agreement exists between these measured values and the predicted values of both the vertically and horizontally polarized noise, with almost all the data being within 1 standard deviation of the predicted values, if the standard deviation is calculated from the values of D_u given in CCIR Report 322 [1]. This result is encouraging, but more data are needed to fully validate the model.

Additional Development of the Model

Several possibilities exist for further developing the model. However, the most useful additional development at present would be to extend the model's frequency range, now 10 to 30 kHz, to 60 kHz, since this extended frequency range is of great interest for the DCA MEECN (Minimum Essential Emergency Communications Network), and of operational concern to the Navy and the Air Force. Such an extension would first require that the COMPWR and NOISLAN programs be modified to accurately predict vertically polarized noise at the earth's surface up to 60 kHz. The validity of the predictions from 30 to 60 kHz could be checked against available measured vertical noise data, just as was done for 10 to 30 kHz. Then, the HORNS program could easily be extended to calculate high-altitude noise. Another improvement in the atmospheric-noise prediction model would be to include the noise contributed by the quasi-TE mode.

COMPUTER-PROGRAM MODEL FOR PREDICTING HORIZONTALLY AND VERTICALLY POLARIZED VLF ATMOSPHERIC RADIO NOISE AT ELEVATED RECEIVERS

INTRODUCTION

The communication effectiveness of any radio circuit or link strongly depends on the signal-to-noise ratio in the receiver. At very low frequencies the propagated radio noise generated by lightning discharges, referred to as atmospheric radio noise, is frequently the dominant system noise.

Many workers have studied atmospheric radio noise [1-7]. In particular, Ref. 3 decribes a computer model which predicts the atmospheric radio noise generated by thunderstorms in all regions of the world and then calculates the propagation of the noise to a receiver at any point on the earth's surface. By performing this computation over a grid of receiver locations, a map of the intensity of atmospheric radio noise can be generated [8] similar to those given in Ref. 1. However, as with most work in this field, only the vertical electric component of the noise is calculated.

In recent years interest has grown in the use of horizontal electric polarized waves for high-altitude air-to-air communication. This polarization should be advantageous, because it is operationally easier for a high-speed high-altitude jet aircraft to trail a horizontal long wire antenna, which preferentially excites transverse electic radio waves, and because the vertically polarized lightning strokes are thought to be less effective in generating horizontally polarized noises. Therefore, a requirement exists to be able to accurately predict these noise levels.

The theoretical and experimental effects associated with horizontally polarized VLF-wave transmission and reception have been treated by several authors [9-13]. Also, the use of airborne, horizontally polarized transmitting and receiving antennas for communications at VLF has been discussed several times [14-17]. The WAVEGUID computer program [11] contains the algorithms necessary to compute all the electric and magnetic components of a propagating electromagnetic wave in a given waveguide mode at any height above the ground, if the vertical electric component of the wave at the ground is known. The COMPWR and NOISLAN programs produced by the Westinghouse Georesearch Laboratory (WGL) [3] calculate the amplitudes of the vertical electric VLF noise in the three dominent quasi-transverse-magnetic (quasi-TM) waveguide modes from each equivalent atmospheric noise source. By combining these programs, NRL has now developed a working computer-program model for predicting the horizontally and vertically polarized atmospheric noise at any location, season, time of day, and altitude. This noise program should assist in the prediction of system performance.

THE PROGRAMS THAT FORM THE NOISE-PREDICTING MODEL

References 3 and 11 give complete descriptions of the COMPWR, NOISLAN, and WAVEGUID programs. The following subsections briefly describe the features of each program essential to the new program.

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Description of COMPWR and NOISLAN

The atmospheric-noise model is built on thunderstorm-day data, from which vertical electrical noise intensity is derived. The thunderstorm-day is a weather statistic which gives over a 30-day interval the number of days during which at least one thunderstorm occurred in the vicinity of the observation site. The data base used by the program represents observations collected over several decades and at thousands of locations. In addition the model uses data on the diurnal variation and variability of thunderstorm occurrences. Algorithms convert thunderstorm days into the number and type of lightning discharges per unit area on the earth's surface. Also, the current moments, waveforms, and power spectral densities of the pulses are calculated. The resultant source powers are combined into equivalent noise "transmitters" representing 5° by 5° areas of the earth's surface. These 5° by 5° areas are further combined into 15° by 15° areas. The program decides, based on the proximity and power of the equivalent noise transmitters, whether to use 5° by 5° or 15° by 15° areas in the final computations.

After COMPWR establishes the location, power, and standard deviation of the power for each equivalent noise transmitter, the NOISLAN program then computes the resulant vertical electric field at the specified receiver location using the propagation equation

$$E_{n,z}^{(i)}(0) = K \sqrt{\frac{P_r^{(i)} \, |\Lambda_{n,t}^{(i)}| \, |\Lambda_{n,r}|}{f h_t^{(i)} h_r a \sin (d^{(i)}/a)}} \; 10^{-\alpha_n^{(i)} d^{(i)}/2 \times 10^7}, \tag{1}$$

where

$E_{n,z}^{(i)}(0)$	is the vertical electric component of the atmospheric noise contribution of the ith noise source in the nth waveguide mode at the surface of the waveguide, in volts per meter,
K	is a normalizing constant equal to 1.58×10^5,
$P_r^{(i)}$	is the vertical effective radiated power (VERP) of the ith equivalent noise transmitter in watts,
$\Lambda_{n,t}^{(i)}$	is the excitation factor of the nth waveguide mode at the ith transmitter locations,
$\Lambda_{n,r}$	is the excitation factor of the nth waveguide mode at the receiver location,
f	is the frequency in hertz,
$h_t^{(i)}$ and h_r	are the effective ionospheric heights in meters at the ith transmitter and at the receiver respectively,
a	is the earth's radius in meters,
$d^{(i)}$	is the path length from the ith transmitter to the receiver in meters,

and

$\alpha_n^{(i)}$	is the effective attenuation rate in decibels per megameter (1000 km) for the nth mode from the ith transmitter.

The effective attenuation rate $\alpha_n^{(i)}$ is computed from the combination of several attenuation terms as follows:

$$\alpha_n^{(i)} = M^{(i)} \left[\alpha_{a,n} + \left(d^{(i)} \right)^{-1} \sum_{l=1}^{L} \Delta\alpha_{I,l,n}^{(i)} \, \Delta d_l \right] + \left(d^{(i)} \right)^{-1} \sum_{k=1}^{K} \Delta\alpha_{G,k,n}^{(i)} \, \Delta d_k. \tag{2}$$

In Eq. (2), $M^{(i)}$ is a directional factor and is given by

$$M^{(i)} = 1 - k \sin \phi_a^{(i)}, \tag{3}$$

where $\phi_a^{(i)}$ is the direction of propagation of the wave from the ith transmitter with respect to geographic north and k is given by

$$k = (10^4/f)^{1/2} \, K_{10\mathrm{kHz}} \, e^{-\beta|\theta - 90|}, \tag{4}$$

in which

θ	is the colatitude of the receiver in degrees,
f	is the frequency in hertz,
K_{10kHz}	is a constant that for west-to-east propagation equals 0.68 during the day and 0.70 at night and for east-to-west propagation equals 1.10 during the day and 2.00 at night,

and

β	is a constant that equals 0.04 during the day and 0.03 at night.

Also in Eq. (2), $\alpha_{a,n}$ is the attenuation rate (day or night) of the nth-order mode over seawater for the isotropic case, $\Delta\alpha_{I,l,n}^{(i)}$ is the differential ionospheric attenuation rate in effect for the incremental distance Δd_l along the path, $\Delta\alpha_{G,k,n}^{(i)}$ is the differential ground attenuation rate due to ground conductivities other than seawater, and Δd_k is the distance segment of the propagation path within a given conductivity area. The differential ionospheric attenuation term is obtained from

$$\Delta\alpha_{I,l,n}^{(i)} = f\,[2A\cos^2\theta_l + B(\sin|\chi_l|)\sin^2\theta_l], \tag{5}$$

where

θ_l	is the colatitude of the lth segment of the propagation path in degrees,
χ_l	is the solar zenith angle of the lth segment of the propagation path,
A	equals 2.2×10^{-5} dB/Mm-Hz,

and

B	equals 2.8×10^{-5} dB/Mm-Hz.

The values in Eq. (2) for $\Delta\alpha_{G,k,n}^{(i)}$ are the differences between the attenuation rate of seawater for the isotropic case $\alpha_{a,n}$ and the attenuation rates for the lower ground conductivities as stated in the equation

$$\Delta\alpha_{G,k,n}^{(i)} = \alpha_{G,k,n}^{(i)} - \alpha_{a,n},$$

where values of $\alpha_{G,k,n}^{(i)}$ and $\alpha_{a,n}$ are those given by Wait and Spies [18] for $h = 70$ km and $\beta = 0.3$ km^{-1} for day and $h = 90$ km and $\beta = 0.5$ km^{-1} for night with $\Omega = 0$ (with their parameter β *not* being the same as the parameter β in Eq. (4)). However, Ref. 18 is incomplete, because no theoretical data were obtained for some of the lower values of ground conductivity for the second-order and third-order modes. When the theoretical data were not available, the authors of the NOISLAN program appear to have extrapolated data according to an unspecified procedure. The values of Δd_k are computed from a digitized version [3, pp. 4-19] of a map of VLF ground conductivity developed for NRL by Morgan [19].

The field-strength contributions from each transmitter and each mode are then summed in an RMS manner, thus giving the vertical electric field strength $E_z(0)$ of the noise at the ground for a specific hour of the day, month, location, and frequency:

$$E_z(0) = \left[\sum_{n=1}^{3} \sum_{i=1}^{N} (E_{n,z}^{(i)})^2\right]^{1/2}. \tag{6}$$

NOISLAN also computes the standard deviation of the noise σ_n and the voltage deviation V_d. However, a discussion of these two parameters is not pertinent to this report.

Additional Options for COMPWR and NOISLAN

Two options have been added to the WGL model. The first involves a change in the part of the WGL model contained in program COMPWR. The original version of COMPWR has been designated

as option I of the COMPWR program, and the modification to be described has been designated as option II. COMPWR computes the average vertical effective radiated power (VERP) and the standard deviation of the VERP for each equivalent noise transmitter for each month of the year. COMPWR then multiplies the average VERP by a diurnal modifier to give the VERP for each hour of the day. Option II involves changes to the diurnal modifiers as well as inclusion of an additional set of modifiers to adjust the average VERP. These changes were developed empirically [20,21].

Another option has been added in the form of a modification to the NOISLAN program. The original version of NOISLAN has been designated as option I, and the new modification has been designated as option II. The modification involves the terms $\Lambda_{n,r}$ and $\Lambda_{n,t}^{(i)}$ from Eq. (1) and $\alpha_{a,n}$ and $\Delta\alpha_{a,k,n}^{(i)}$ from Eq. (2). In option I of the NOISLAN program the values for these terms are those given by Watt [22], who in turn derived the values from the work of Wait and Spies [18]. However, the computer program used to generate the data of Ref. 18 (in many cases) could find no solution for the second-order and third-order modes, especially for the lower values of ground conductivity. Further, the accuracy of Wait's second-order-mode and third-order-mode roots has been questioned by Pappert et al. [23]. Because of these problems, it was decided to recompute the appropriate mode parameters using the WAVEGUID program. These parameters were then used to compute the aforementioned terms from Eqs. (1) and (2). The values of the propagation parameters computed from the WAVEGUID program agreed with the values used in the NOISLAN program for the first-order mode, but disagreed (as noted by Pappert [23]) for the second-order and third-order modes. Since the second-order and third-order modes often carry more horizontally polarized energy than does the first-order mode, accurate computation of the higher order modes is important in the prediction of high-altitude atmospheric radio noise. The WAVEGUID mode-parameter solutions are believed to be more accurate than those of Refs. 18 and 22. Option II, therefore, has been included in NOISLAN to use the newly calculated parameters. Also there has been interest in extending the VLF atmospheric noise model upward in frequency to cover the lower end of the LF spectrum (30 to 60 kHz). This extension would not be possible using the results of Refs. 18 and 22 but would be possible by use of the WAVEGUID program.

Description of WAVEGUID

The WAVEGUID computer program was originally developed at the Naval Electronics Laboratory Center to predict the vertical electric field received at a point on the earth's surface produced by a vertical electric transmitting antenna at another point on the earth's surface. It was later extended to give the crosspolarized (horizontal) components of the field at any height in the earth-ionosphere waveguide and even in the ionosphere itself [11, 24, 25]. The basic formulation of the equations for the fields are given in Ref. 13. The net resultant vertical electric field at the earth's surface is represented as a summation of waveguide-mode fields:

$$E_z(0) = \frac{i\sqrt{\mu_0/\epsilon_0}}{\sqrt{d\lambda\, h}}\, Ids \sqrt{\frac{d/a}{\sin(d/a)}}\, \sum_n [\sin^{3/2}\theta_n\, \Lambda_n \exp(i\pi/4 + ik_0 d \sin\theta_n)], \tag{7}$$

where

θ_n	is the eigenangle of the nth waveguide mode,
Ids	is the dipole moment (ampere-meter) of the vertical antenna located on the earth's surface,
Λ_n	is the excitation factor of the nth waveguide mode,
a	is the radius of the earth,
d	is the great-circle distance between the transmitter and the receiver,
h	is the reference height of the ionosphere (used in the definition of Λ_n),
k_0	$= \omega/c$ is the free-space propagation constant of the wave,
λ	is the wavelength of the wave,

and

μ_0 and ϵ_0 are the magnetic permeability and dielectric constant of free space.

Each waveguide mode is characterized by its own value of attenuation rate α_n and phase velocity v_p^n, which are related to the eigenangle of the nth waveguide mode θ_n according to

$$\alpha_n = 0.02895\, \omega\, \mathrm{Im}(\sin\theta_n) \tag{8}$$

and

$$v_p^n = \frac{c}{\mathrm{Re}(\sin\theta_n)}, \tag{9}$$

where α_n is measured in decibels per 1000 km (dB/Mm) and c is the speed of light.

The values of the waveguide-mode eigenangles depend on the reflection coefficients of the ground and the ionosphere. Because of its anisotropy, the ionosphere has four reflection coefficients ${}_{\parallel}R_{\parallel}(\theta_n)$, ${}_{\perp}R_{\perp}(\theta_n)$, ${}_{\parallel}R_{\perp}(\theta_n)$, and ${}_{\perp}R_{\parallel}(\theta_n)$ for a given angle of incidence θ_n. The ground is assumed to be isotropic and to have reflection coefficients ${}_{\parallel}\bar{R}_{\parallel}(\theta_n)$ and ${}_{\perp}\bar{R}_{\perp}(\theta_n)$, which are calculable from the eigenangle, ground conductivity, and dielectric constant. The WAVEGUID program uses a procedure for calculating the ground and ionospheric reflection coefficients referenced to any height z in the waveguide. References 26, 27, and 28 discuss the reflection coefficients further. The eigenangle θ_n for a given waveguide mode is obtained by satisfying the mode equation

$$0 = \left\| \begin{bmatrix} {}_{\parallel}R_{\parallel}(\theta_n) & {}_{\perp}R_{\parallel}(\theta_n) \\ {}_{\parallel}R_{\perp}(\theta_n) & {}_{\perp}R_{\perp}(\theta_n) \end{bmatrix} \begin{bmatrix} {}_{\parallel}\bar{R}_{\parallel}(\theta_n) & 0 \\ 0 & {}_{\perp}\bar{R}_{\perp}(\theta_n) \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\|. \tag{10}$$

This matrix equation is simply rewritten

$$F(\theta_n) = ({}_{\parallel}R_{\parallel}\,{}_{\parallel}\bar{R}_{\parallel} - 1)\,({}_{\perp}R_{\perp}\,{}_{\perp}\bar{R}_{\perp} - 1) - {}_{\perp}R_{\parallel}\,{}_{\perp}\bar{R}_{\perp}\,{}_{\parallel}R_{\perp}\,{}_{\parallel}\bar{R}_{\parallel} = 0, \tag{11}$$

where the dependence of the reflection coefficient on θ_n has been suppressed to simplify the notation. The new variable $F(\theta_n)$ is defined also. Each value of θ_n which satisfies Eq. (11) is the eigenangle of the nth waveguide mode. The excitation factor Λ_n for the nth waveguide mode is obtained from

$$\Lambda_n = -i\,\frac{kh}{2}\sin\theta_n \frac{(1 + {}_{\parallel}\bar{R}_{\parallel})^2\,(1 - {}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp})}{{}_{\parallel}\bar{R}_{\parallel}\left.\dfrac{\partial F(\theta)}{\partial\theta}\right|_{\theta=\theta_n}}, \tag{12}$$

where the variable $F(\theta)$ is as defined in Eq. (11). In addition to the vertical electric field E_z for a given waveguide mode having eigenangle θ_n, in general there exist five other nonzero field components E_x, E_y, H_x, H_y, and H_z within the waveguide for each mode.

These extra fields arise because the ionosphere is anisotropic on account of the earth's magnetic field. Propagation through and reflection from such an anisotropic medium rotates the plane of polarization of the incident wave to generate crosspolarized reflected fields. Alternatively one could consider the linearly polarized wave transmitted from the vertical electric dipole antenna to be a linear superposition of right-handed and left-handed circularly polarized waves, each reflected with a different amplitude at the ionosphere. The ionospheric reflection causes a mixture of wave polarizations at a reception point on or above the earth's surface. The fields at height z are related to the vertical electric fields at the earth's surface $E_z(0)$ in a given waveguide mode by the following functions of reflection coefficients and Hankel functions:

$$E_z(z)/E_z(0) = f_{\parallel}(z), \tag{13}$$

$$E_x(z)/E_z(0) = g(z)/S, \tag{14}$$

$$E_y(z)/E_z(0) = -\frac{{}_{\parallel}R_{\perp}\,(1 + {}_{\perp}\bar{R}_{\perp})\,f_{\perp}(z)}{S(1 + {}_{\parallel}\bar{R}_{\parallel})\,(1 - {}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp})}, \tag{15}$$

$$H_z(z)/E_z(0) = -\frac{{}_{\parallel}R_{\perp}(1+{}_{\perp}\bar{R}_{\perp})\,f_{\perp}(z)}{\eta(1+{}_{\parallel}\bar{R}_{\parallel})\,(1-{}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp})}, \tag{16}$$

$$H_y(z)/E_z(0) = \frac{f_{\parallel}(z)}{\eta\, S}, \tag{17}$$

and

$$H_x(z)/E_z(0) = \frac{{}_{\parallel}R_{\perp}(1+{}_{\perp}\bar{R}_{\perp})}{i\,\eta\,k\,S(1+{}_{\parallel}\bar{R}_{\parallel})\,(1-{}_{\perp}\bar{R}_{\perp}\,{}_{\perp}R_{\perp})}\;\frac{df_{\perp}(z)}{dz}, \tag{18}$$

where

$$f_{\parallel}(z) = \exp\left[\frac{z-D}{a}\right]\;\frac{F_1\,h_1(q)+F_2\,h_2(q)}{F_1\,h_1(q_d)+F_2\,h_2(q_d)}, \tag{19}$$

$$f_{\perp}(z) = \frac{F_3\,h_1(q)+F_4\,h_2(q)}{F_3\,h_1(q_d)+F_4\,h_2(q_d)}, \tag{20}$$

and

$$g(z) = \frac{1}{ik}\,\frac{d}{dz}\,[f_{\parallel}(z)], \tag{21}$$

in which

$$F_1 = -\left[H_2(q_0) - i\,\frac{n_0^2}{N_g^2}\left[\frac{ak}{2}\right]^{1/3}(N_g^2-S^2)^{1/2}\,h_2(q_0)\right], \tag{22}$$

$$F_2 = H_1(q_0) - i\,\frac{n_0^2}{N_g^2}\left[\frac{ak}{2}\right]^{1/3}(N_g^2-S^2)^{1/2}\,h_1(q_0), \tag{23}$$

$$F_3 = -\left[h'_2(q_0) - i\left[\frac{ak}{2}\right]^{1/3}(N_g^2-S^2)^{1/2}\,h_2(q_0)\right], \tag{24}$$

$$F_4 = h'_1(q_0) - i\left[\frac{ak}{2}\right]^{1/3}(N_g^2-S^2)^{1/2}\,h_1(q_0), \tag{25}$$

$$q = \left[\frac{2}{ak}\right]^{-2/3}\left[C^2 - \frac{2}{a}(h-z)\right], \tag{26}$$

and

$$q_d = \left[\frac{2}{ak}\right]^{-2/3}\left[C^2 - \frac{2}{a}(h-D)\right], \tag{27}$$

with

$$q_0 = \left[\frac{2}{ak}\right]^{-2/3}\left[C^2 - \frac{2h}{a}\right], \tag{28}$$

$$H_j\,(q) = h_j(q) + \frac{1}{2}\left[\frac{2}{ak}\right]^{2/3} h_j(q), \quad j = 1, 2, \tag{29}$$

$$n^2 = 1 - \frac{2}{a}(h-z), \tag{30}$$

$$n_0^2 = 1 - \frac{2}{a}\,h, \tag{31}$$

and

$$N_g^2 = \epsilon/\epsilon_0 - j\,\frac{\sigma}{\omega\epsilon}. \tag{32}$$

In these expressions

a	is the radius of the earth,
z	is the height of the receiver above ground,
N_g	is the index of refraction of the ground surface,
D	is the reference altitude in the waveguide for evaluating the mode parameters and the reflection coefficients,
k	is the propagation constant of the waves in free space ($= \omega/c$),
S and C	are the sine and cosine of the eigenangle θ_n,
$h_1(q)$ and $h_2(q)$	are the modified Hankel functions of order 1/3,

and

η	is the impedance of free space.

Using these relations, one can obtain a desired field of a given mode at any altitude z, if one knows the vertical field in that mode at the earth's surface. The NOISLAN prediction program calculates the vertical noise fields in the three strongest quasi-TM waveguide modes in making its noise prediction. So, the values of $E_z(0)$ for the three most important modes are available, and one can obtain the crosspolarized fields at any height z by applying Eqs. (13) through (18) to the NOISLAN $E_z(0)$ values.

HORNS Program and Approximations

The HORNS portion of the atmospheric-noise model predicts high-altitude atmospheric noise at any polarization by using the functions of the WAVEGUID program, as summarized in Eqs. (13) through (18), and the values of $E_z(0)$ computed by the NOISLAN program. Equations (13) through (18) may be rewritten in a notation more appropriate to the summation of fields from many sources:

$$E_{n,z}^{(i)}(z) = f_{\parallel,n}^{(i)}(z)\; E_{n,z}^{(i)}(0), \tag{13'}$$

$$E_{n,x}^{(i)}(z) = g_n^{(i)}(z) E_{n,z}^{(i)}(0)/S_n^{(i)} \tag{14'}$$

$$E_{n,y}^{(i)}(z) = \frac{-{}_{\parallel}R_{\perp,n}^{(i)}(1 + {}_{\perp}\bar{R}_{\perp,n}^{(i)}) f_{\perp,n}^{(i)}(z) E_{n,z}^{(i)}(0)}{S_n^{(i)}\,(1 + {}_{\parallel}\bar{R}_{\parallel,n}^{(i)})\,(1 - {}_{\perp}\bar{R}_{\perp,n\perp}^{(i)} R_{\perp,n}^{(i)})}, \tag{15'}$$

$$H_{n,z}^{(i)}(z) = \frac{-{}_{\parallel}R_{\perp,n}^{(i)}(1 + {}_{\perp}\bar{R}_{\perp,n}^{(i)}) f_{\perp,n}^{(i)}(z) E_{n,z}^{(i)}(0)}{\eta(1 + {}_{\parallel}\bar{R}_{\parallel,n}^{(i)})(1 - {}_{\perp}\bar{R}_{\perp,n\perp}^{(i)} R_{\perp,n}^{(i)})}, \tag{16'}$$

$$H_{n,y}^{(i)}(z) = -\frac{f_{\parallel,n}^{(i)}(z)\; E_{n,z}^{(i)}(0)}{\eta\; S_n^{(i)}}, \tag{17'}$$

and

$$H_{n,x}^{(i)}(z) = \frac{{}_{\parallel}R_{\perp,n}^{(i)}(1 + {}_{\perp}\bar{R}_{\perp,n}^{(i)})}{i\eta k S_n^{(i)}(1 + {}_{\parallel}\bar{R}_{\parallel,n}^{(i)})(1 - {}_{\perp}\bar{R}_{\perp,n}^{(i)}\,{}_{\perp}R_{\perp,n}^{(i)})}\;\frac{df_{\perp,n}^{(i)}(z)}{dz}\; E_{n,z}^{(i)}(0). \tag{18'}$$

In these expressions the field symbols such as $E_{n,z}^{(i)}(z)$ refer to the field strength produced by the ith noise source in the nth waveguide mode. The symbols for the height gain functions, such as $f_{\parallel,n}^{(i)}(z)$, refer to these functions in the nth waveguide mode and from the ith transmitter. Likewise the ground and ionospheric reflection coefficients refer to particular modes and to waves from a particular noise source.

After the individual contributions to the components of the noise field have been calculated, the vertically polarized electric and magnetic noise fields may be obtained by analogy to Eq. (6):

$$E_z(z) = \left\{ \sum_{i=1}^{N} \sum_{n=1}^{3} |E_{n,z}^{(i)}(z)|^2 \right\}^{1/2} \tag{33}$$

and

$$H_z(z) = \left\{ \sum_{i=1}^{N} \sum_{n=1}^{3} |H_{n,z}^{(i)}(z)|^2 \right\}^{1/2}. \tag{34}$$

A random-phase approximation has been made in deriving Eqs. (33) and (34). In other words, no correlations are assumed among the elements $E_{n,z}^{(i)}$ in the sum of Eq. (33) or among the elements $H_{n,z}^{(i)}$ of Eq. (34). For different values of i (different sources) the random-phase approximation is undoubtedly correct, since there is no correlation in the arrival time of energy from different sources. However, for different values of n (different modes from the same source) the random-phase approximation is less accurate, since some correlation exists among the arrival times of different modes from the same source. However, since the noise "transmitter" locations are only the noise power centroids for noise sources in their respective areas and not the noise sources themselves, the actual propagation paths from each lightning discharge to the receiver are not known. Thus one cannot accurately calculate the phase angles of the various modes, leaving the random-phase approximation as the best alternative.

To obtain equations for the horizontally polarized noise fields, one must first select a horizontal axis direction x' along which the noise is to be specified. The previously used coordinates x, y, and z are defined in relation to the direction of propagation of a wave: z is the vertical direction, x is horizontal in the direction of propagation of the wave, and y is horizontal perpendicular to the direction of wave propagation. Since the equivalent sources are at different bearings from the receiving location, one cannot simply add the $H_{n,x}^{(i)}(z)$ field to that from another source such as $H_{n,x}^{(i+1)}(z)$, because they will not be parallel. Instead one must find the contribution of the various fields to a resultant component fixed in space using the normal processes of vector resolution and vector addition. One then obtains the total horizontal fields in the fixed x' directions as follows:

$$E_{x'}(z) = \left\{ \sum_{i=1}^{N} \sum_{n=1}^{3} \left| E_{n,x'}^{(i)}(z) \cos\theta_{x,x'}^{(i)} + E_{n,y}^{(i)}(z) \sin\theta_{x,x'}^{(i)} \right|^2 \right\}^{1/2} \tag{35}$$

and

$$H_{x'}(z) = \left\{ \sum_{i=1}^{N} \sum_{n=1}^{3} \left| H_{n,x}^{(i)}(z) \cos\theta_{x,x'}^{(i)} + H_{n,y}^{(i)}(z) \sin\theta_{x,x'}^{(i)} \right|^2 \right\}^{1/2}. \tag{36}$$

The resultant noise predictions $E_{x'}(z)$ and $H_{x'}(z)$ are real numbers, and the waveguide-mode field amplitudes $E_{n,x}^{(i)}$, $E_{n,y}^{(i)}$, $H_{n,x}^{(i)}$, and $H_{n,y}^{(i)}$ are complex phasors. Hence one must use the absolute magnitudes of these contributions in Eqs. (35) and (36).

To determine $H_{n,z}^{(i)}(z)$ from $E_{n,z}^{(i)}(0)$ according to Eq. (16′), one needs knowledge of $\theta_n^{(i)}$, the eigenangle of the nth waveguide mode that carries the wave from the ith noise transmitter, and of the ionospheric and ground reflection coefficients for this propagating mode. How can one obtain $\theta_n^{(i)}$? One could determine $\theta_n^{(i)}$ from the WAVEGUID program using an appropriate anisotropic ionosphere model by iteratively solving Eq. (11), as is normally done with this program. However, this procedure is time consuming when applied to noise predictions, because it ordinarily requires several iterations to settle on a satisfactory value of θ_n. This undertaking could become expensive and tedious, since there may be as many as 70 significant thunderstorm atmospheric noise sources, each having a different bear-

ing from the receiver location. Since the angle of arrival of energy from each source would be different, a new calculation of θ_n would be required for each case. An alternative method has been used in the present version of the program.

The approximation for θ_n in the present program is to substitute the eigenangle θ_n which has already been calculated for the quasi-TM mode with an isotropic ionosphere. This procedure makes the assumption that the field expressions in Eqs. (13′) thorugh (18′) are not sensitive to the value θ_n . This assumption was tested and partially verified for a sample of noise predictions. Certainly this method of selecting θ_n is arbitrary, and it might be improved in subsequent versions of the program. Once θ_n is selected, it is used for the evaluation of all the factors that enter into the computation of the elevated fields. The geomagnetic field strength and direction at the receiver location is calculated using a dipole model of the earth's field. The ith azimuth angle required to evaluate the ionospheric reflection coefficients is evaluated using this geomagnetic model and a knowledge of the direction to the ith equivalent noise source.

The atmospheric-noise model calculates the high-altitude electromagnetic fields propagated in three quasi-TM modes in the earth-ionosphere waveguide. The fields generated by the vertical component of the lightning stroke but propagated in the quasi-transverse-electric (quasi-TE) modes are ignored. This deficiency of the prediction model may be remedied in future versions. It seems reasonable that the contributions by the quasi-TE mode to the horizontal electric and vertical magnetic field should be comparable to the contributions by the quasi-TM mode; hence the observed noise might exceed the predicted values by 3 dB.

The atmospheric-noise model also ignores the contributions of the horizontal component of the lightning discharge, because it has been assumed that the horizontal discharges are much weaker than the vertical discharges. Recent detailed analyses of lightning discharge paths [29] suggest that a substantial elevated horizontal component of the discharge channel existed in several thoroughly analyzed cloud-to-ground discharges. This strong horizontal component would be inconsistent with our present noise model and suggests an important topic for further investigation. A detailed statistical study of lightning-channel structures and of received noise polarizations would be useful in assessing this question.

COMPARISON OF PREDICTIONS WITH OBSERVATIONS

The predictions of the vertical noise fields on the ground obtained from COMPWR and NOISLAN can be readily compared with data. From 1958 through 1967, at first the National Bureau of Standards (NBS) and after a reorganization the Environmental Science Services Administration (ESSA) supervised a worldwide network of atmospheric-radio-noise recording stations [30]. Measured long-term noise data at 13 kHz from 12 of these stations have been compared with COMPWR/NOISLAN predictions. Figures 1 and 2 show a comparison between the NBS experimental data and the predictions using option II of COMPWR and options I and II of NOISLAN.

Whereas many data exist for vertically polarized atmospheric electric noise, few data are available for vertically polarized magnetic noise. The Mitre Corporation has made a few nearly simultaneous measurements of the vertically and horizontally polarized magnetic noise fields at high altitudes using loop antennas. The Mitre Corporation personnel used a horizontal loop antenna extended behind a jet aircraft in measuring the vertical magnetic field strength. The receiver used for the measurements was an RCA VLF amplitude-probability-distribution receiver loaned by NRL for the experiments. A pair of orthogonal-crossed-loop antennas were used in measuring the horizontal magnetic field. The experimenters state (G. Hirst, private communication) that their crossed-loop system did not have an omnidirectional reception pattern, because of induced coupling from the aircraft structure. The crossed-loop antennas were coupled to produce a response to elliptically polarized horizontal magnetic noise fields. Neither the directions of the major and minor axes of the ellipse nor the direction of rotation of the

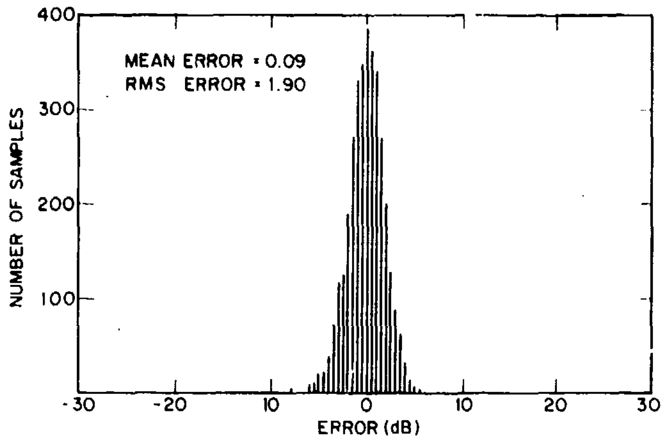

Fig. 1 — Histograms of errors (observed values minus predicted values) in predicting vertically polarized electric noise at 13 kHz using NOISLAN option I and COMPWR option II

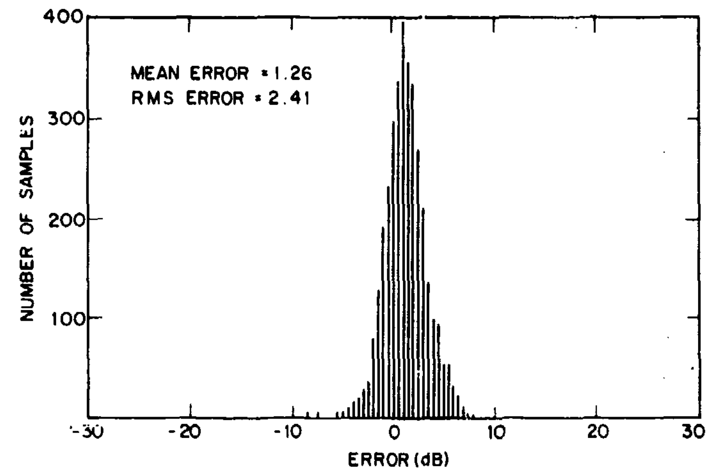

Fig. 2 — Histogram of errors (observed values minus predicted values) in predicting vertically polarized electric noise at 13 kHz using NOISLAN option II and COMPWR option II.

elliptically polarized waves are available. These parameters are important in comparing the data with detailed theoretical predictions from the atmospheric-noise program. They were not important to the Mitre measurement program. In spite of these difficulties, the Mitre data are valuable at present, because they are practically all the data there are. The voltage outputs of the crossed-loop antennas were added together with a 90° phase shift, as is normally done in a crossed-loop system. The resultant field strengths were then normalized to the value of the vertical electric field that an equivalent TM propagation wave would possess to give the observed magnetic-field value. These are the values of E_z presented in Figs. 3 through 8 and in Table 1. The values of $\mathscr{H}_z$, where $\mathscr{H}_z = \eta H_z$, with η being the impedance of free space, were those measured on the horizontal loop antenna.

From Figs. 3 through 8 and from Table 1, one can note that the equivalent vertical electric noise field E_z is predicted to be relatively constant from day to night and that it exceeds the equivalent vertical magnetic noise field $\mathscr{H}_z$ by about 30 dB during the day and by about 15 dB during the night. The smaller predicted value of $\mathscr{H}_z$ during the day is directly related to the smaller value of the ionospheric reflection coefficient $_{\parallel}R_{\perp}$ that describes the change of wave polarization from vertical electric to vertical magnetic (Eq. (16′)).

An interesting case of disagreement with the model is observed in Fig. 4, where the noise data, taken immediately after sunrise at the receiver, have a separation of $\mathscr{H}_z$ from E_z more characteristic of nighttime than daytime conditions. Such a case could occur if about 50% of the noise sources were in

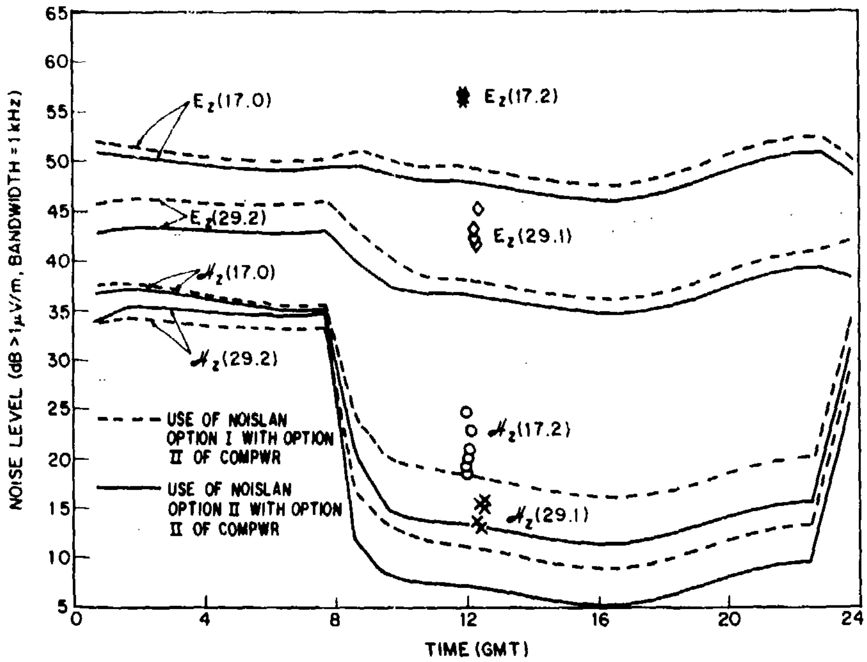

Fig. 3 — Predicted values (lines) and observed values (symbols) of the E_z and H_z components of RMS atmospheric noise at a 10-km altitude at 47°N, 55°W on July 15, 1976. The numbers in parentheses give the frequency in kilohertz.

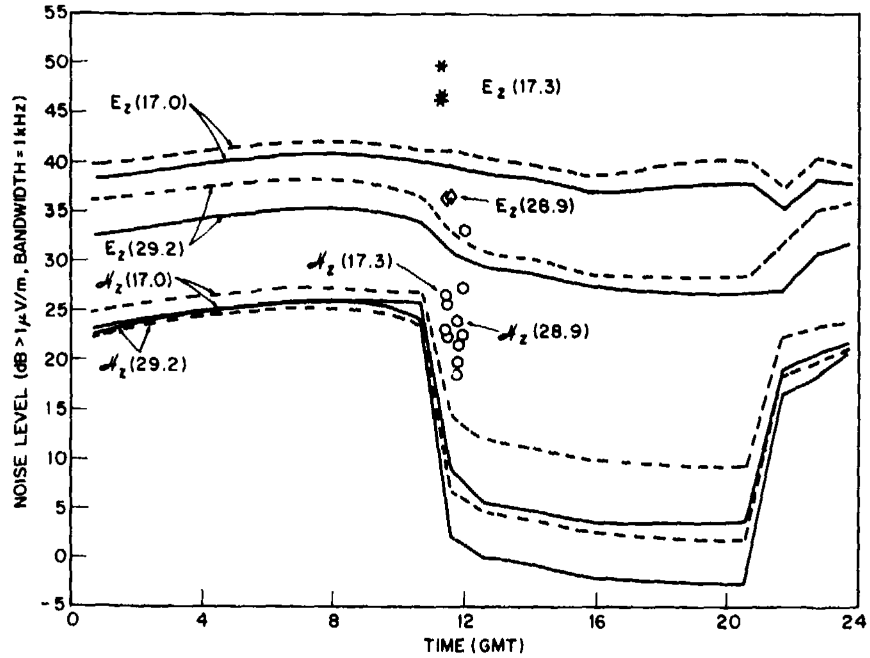

Fig. 4 — Predicted and observed values of RMS atmospheric noise at a 10-km altitude at 44°N, 65°W on February 20, 1976

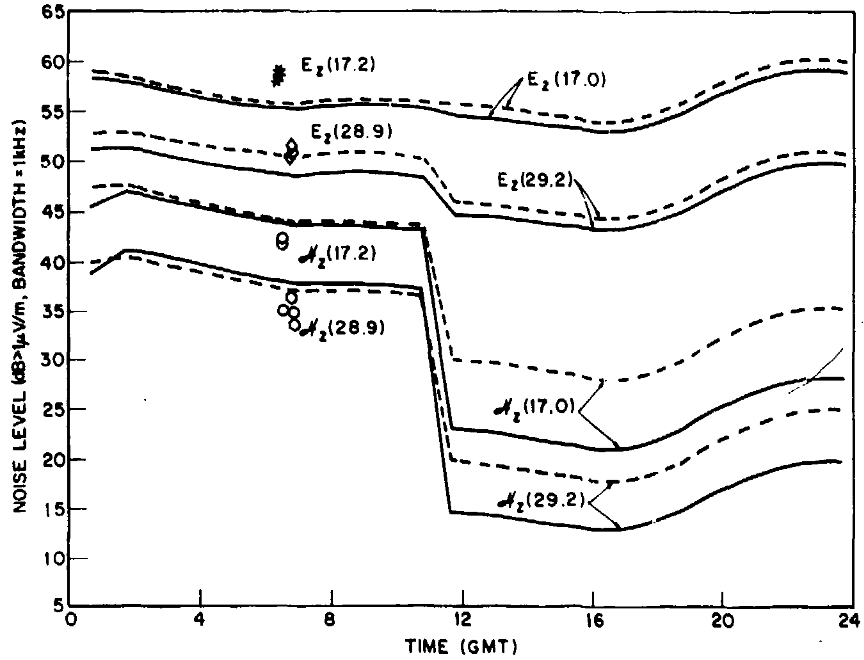

Fig. 5 — Predicted and observed value of RMS atmospheric noise at a 10-km altitude at 38°N, 82°W on August 17, 1976

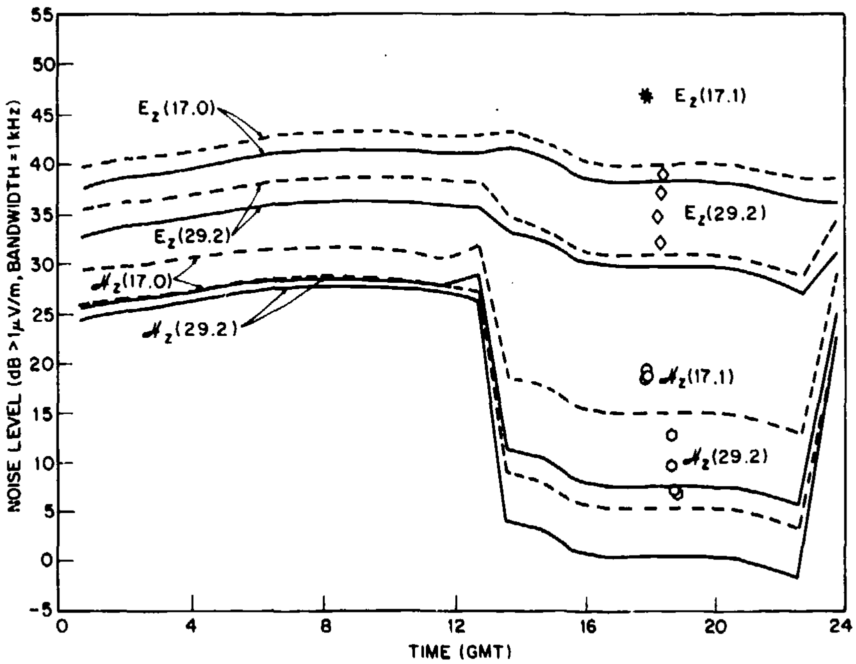

Fig. 6 — Predicted and observed values of RMS atmospheric noise at a 10-km altitude at 39°N, 90°W on January 23, 1976

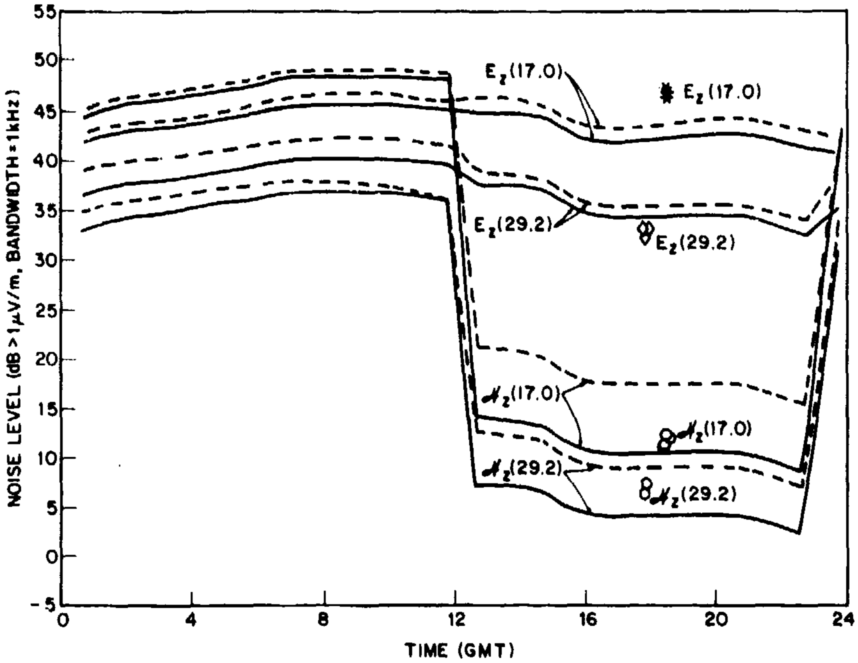

Fig. 7 — Predicted and observed values of RMS atmospheric noise at a 10-km altitude at 28°N, 80°W on February 3, 1976

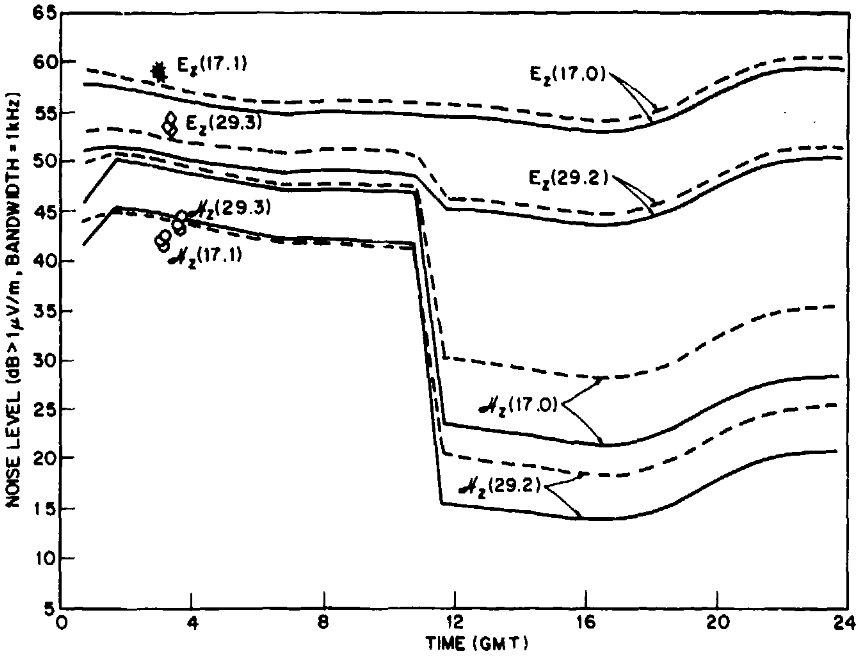

Fig. 8 — Predicted and observed values of RMS atmospheric noise at a 10-km altitude at 38°N, 84°W on August 17, 1976

Table 1 — Summary of Predicted and Observed Separations of RMS Noise as Shown in Figs. 3 thorugh 8

Location	$E_z - H_z$ RMS-Noise Separation (dB)											
	Daytime						Nighttime					
	≈ 17 kHz			≈ 29 kHz			≈ 17 kHz			≈ 29 kHz		
	Opt. I	Opt.II	Obs.	Opt. I	Opt.II	Obs.	Opt. I	Opt.II	Obs.	Opt. I	Opt.II	Obs.
47°N, 55°W	31.0	34.5	35.6	27.0	29.5	28.4	14.0	13.5	—	12.5	6.5	—
44°N, 65°W	29.0	34.5	—	26.5	29.5	—	15.0	15.0	23.2*	13.5	9.5	13.0*
38°N, 82°W	26.0	32.0	—	26.5	30.5	—	12.0	12.0	18.1	13.5	11.0	16.1
39°N, 90°W	25.0	31.0	28.2	26.0	29.5	26.6	12.0	13.0	—	10.0	9.0	—
28°N, 80°W	26.5	32.0	35.4	26.5	30.0	26.2	−2.5	−3.0	—	4.0	3.0	—
38°N, 84°W	26.0	32.0	—	26.0	30.0	—	8.0	8.0	16.9	8.5	7.0	10.2

*Measurements taken during the transition from night to day.

the nighttime portion of the globe and if the effect of passage of the nighttime quasi-TM waves through the terminator region did not suppress the $\mathscr{H}_z$ fields but simply propagated them forward in quasi-TE modes. A future improved version of the noise prediction model may be constructed to take this quasi-TE conversion into account.

In general the measured data are several dB higher than the predictions. However, almost all the E_z measurements are within 1 standard deviation of the predicted levels based on the values of D_u given in CCIR Report 322 [1]. Table 1 presents a summary of predicted and observed $E_z - \mathscr{H}_z$ noise separations. The use of option II of the NOISLAN program with option II of COMPWR yields better predictions of $E_z - \mathscr{H}_z$ noise separation for daytime at ≈17 kHz than does the use of option I of NOISLAN with option II of COMPWR. Other than for daytime at ≈17 kHz the predictions are not better from use of option II of NOISLAN rather than of option I. For both options the predictions for daytime fit the data quite well but the predictions for nighttime do not fit so well. The results are encouraging, but more noise data are needed to fully determine the accuracy of the model.

ADDITIONAL DEVELOPMENT OF THE MODEL

Several possibilities exist for further development of the atmospheric-noise model. Incorporation of TE-mode propagation with conversion at the terminator may be a helpful addition. Also, the extension of the model's frequency range up to 60 kHz would be useful, since the range from 30 to 60 kHz not currently covered is of operational concern. Such an extension would first require a modification of the COMPWR and NOISLAN programs to predict TM noise up to 60 kHz. The propagation parameters could be found using the WAVEGUID program with an appropriate ionospheric model, just the way option II of NOISLAN was prepared. The validity of the extended NOISLAN model could be checked against 51-kHz vertical noise data collected by NBS [30]. In preparation for this comparison effort, the available 51-kHz NBS noise data have been digitized. Results of the analysis may be published in the future.

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